# Chapter 1: Problem Solving

## EXCURSION EXERCISES, SECTION 1.1

1.

| | | |
|---|---|---|
| 2 | 1 | 3 |
| 3 | 2 | 1 |
| 1 | 3 | 2 |

2.

| | | |
|---|---|---|
| 2 | 3 | 1 |
| 3 | 1 | 2 |
| 1 | 2 | 3 |

3.

| | | | |
|---|---|---|---|
| 2 | 1 | 4 | 3 |
| 1 | 3 | 2 | 4 |
| 3 | 4 | 1 | 2 |
| 4 | 2 | 3 | 1 |

4.

| | | | |
|---|---|---|---|
| 1 | 2 | 4 | 3 |
| 3 | 1 | 2 | 4 |
| 2 | 4 | 3 | 1 |
| 4 | 3 | 1 | 2 |

5.

| | | | | |
|---|---|---|---|---|
| 2 | 5 | 1 | 4 | 3 |
| 4 | 1 | 2 | 3 | 5 |
| 3 | 2 | 5 | 1 | 4 |
| 1 | 3 | 4 | 5 | 2 |
| 5 | 4 | 3 | 2 | 1 |

6.

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 1 | 4 | 2 |
| 4 | 3 | 5 | 2 | 1 |
| 5 | 4 | 2 | 1 | 3 |
| 2 | 1 | 3 | 5 | 4 |
| 1 | 2 | 4 | 3 | 5 |

**EXERCISE SET 1.1**

1. 28. Add 4 to obtain the next number.

2. 41. Add 6 to obtain the next number.

3. 45. Add 2 more than the integer added to the previous integer.

4. 216. The numbers are the cubes of consecutive integers. $6^3 = 216$.

5. 64. The numbers are the squares of consecutive integers. $8^2 = 64$.

6. 35. Subtract 1 less than the integer subtracted from the previous integer.

7. $\frac{15}{17}$. Add 2 to the numerator and denominator.

8. $\frac{7}{8}$. Add 1 to the numerator and denominator.

9. $-13$. Use the pattern of adding 5, then subtracting 10 to obtain the next pair of numbers.

10. 51. Add 16 to 35. Add 4 to 1, 7 to 5, 10 to 12, etc., increasing the difference by 3 each time.

11. Correct.

12. Correct.

13. Correct.

14. Incorrect. The sum of two odd counting numbers is always an even counting number.

15. Incorrect. The resulting number will be 3 times the original number.

16. Correct.

17. a. 8 – 0 = 8 cm
    b. 32 – 8 = 24 cm
    c. 72 – 32 = 40 cm
    d. 128 – 72 = 56 cm
    e. 200 – 128 = 72 cm

18. a. 6.5 – 0 = 6.5 cm
    b. 26.0 – 6.5 = 19.5 cm
    c. 58.5 – 26.0 = 32.5 cm
    d. 104.0 – 58.5 = 45.5 cm
    e. 162.5 – 104.0 = 58.5 cm

19. a. 8 cm = 1 unit
    Therfore, 8 · n = 1 · n
    24 cm = 8 · 3 cm
    1 · 3 = 3 units
    b. 40 cm = 8 · 5
    1 · 5 = 5 units
    c. 56 cm = 8 · 7 cm
    1 · 7 = 7 units
    d. 72 cm = 8 · 9 cm
    1 · 9 = 9 units

20. a. 6.5 cm = 1 units
    Therefore, 6.5n = 1 · n
    19.5 cm = 6.5 · 3
    1 · 3 = 3 units
    b. 32.5 cm = 6.5 · 5
    1 · 5 = 5 units
    c. 45.5 cm = 6.5 · 7
    1 · 7 = 7 units
    d. 58.5 cm = 6.5 · 9
    1 · 9 = 9 units

21. It appears that doubling the ball's time, quadruples the ball's distance. In the inclined plane time distance table, the ball's time of 2 seconds has a distance that is quadrupled the ball's distance of 1 second. The ball's time of 4 seconds has a distance that is quadrupled the ball's distance of 2 seconds.

22. It appears that tripling the ball's time, multiplies the ball's distance by a factor of 9. In the inclined plane time distance table, the ball's time of 3 seconds has a distance that is 9 times the ball's distance of 1 second. The ball's time of 9 seconds has a distance that is 9 times the ball's time of 3 seconds.

.

23. 288 cm. The ball rolls 72 cm in 3 seconds. So in doubling 3 seconds to 6 seconds, we quadruple 72 get 288.

24. 18 cm. The ball rolls 8 cm in 1 second and 32 cm in 2 seconds. Therefore, for 1.5 seconds, it would be 8(2) = 16 cm.

25. This argument reaches a conclusion based on a specific example, so it is an example of inductive reasoning.

26. The conclusion is a specific case of a general assumption, so this argument is an example of deductive reasoning.

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$$\begin{aligned}1^3+5^3+3^3&=1+125+27\\&=153\end{aligned}$$

30. This argument reaches a conclusion based on specific examples, so it is an example of inductive reasoning.

31. This argument reaches a conclusion based on a specific example, so it is an example of inductive reasoning.

32. This argument reaches a conclusion based on a specific example, so it is an example of inductive reasoning.

33. Any number less than or equal to $-1$ or between 0 and 1 will provide a counterexample.

34. Any negative number will provide a counterexample.

35. Any number less than $-1$ or between 0 and 1 will provide a counterexample.

36. Any negative number will provide a counterexample.

37. Any negative number will provide a counterexample.

38. $x = 1$ provides a counterexample.

39. Consider any two odd numbers. Their sum is even, but their product is odd.

40. Some even numbers are the product of an odd number and an even number. For example, $2 \times 3 = 6$, which is even, but 3 is odd.

41.

| | | | |
|---|---|---|---|
| **16** | **3** | 2 | 13 |
| **5** | 10 | 11 | **8** |
| **9** | 6 | **7** | 12 |
| 4 | 15 | **14** | 1 |

42.

| | | | | |
|---|---|---|---|---|
| 11 | **24** | 7 | **20** | 3 |
| 4 | **12** | **25** | 8 | 16 |
| **17** | 5 | 13 | **21** | **9** |
| 10 | 18 | 1 | **14** | 22 |
| 23 | 6 | **19** | 2 | 15 |

43. Using deductive reasoning:

| | |
|---|---|
| $n$ | pick a number |
| $6n$ | multiply by 6 |
| $6n+8$ | add 8 |
| $\frac{6n+8}{2}=3n+4$ | divide by 2 |
| $3n+4-2n=n+4$ | subtract twice the original number |
| $n+4-4=n$ | subtract 4 |

44. Using deductive reasoning:

| | |
|---|---|
| $n$ | pick a number |
| $n+4$ | add 4 |
| $3(n+4)=3n+12$ | multiply by 3 |
| $3n+12-7=3n+5$ | subtract 7 |
| $3n+5-3n=5$ | subtract triple the original number |

45.

| | Util | Auto | Tech | Oil |
|---|---|---|---|---|
| A | Xa | Xa | ✓ | Xc |
| T | Xa | Xa | Xc | ✓ |
| M | ✓ | Xb | Xb | Xb |
| J | Xb | ✓ | Xb | Xb |

46.

| | Soup | Entrée | Salad | Dessert |
|---|---|---|---|---|
| C | Xb | ✓ | Xc | Xb |
| S | Xb | Xc | ✓ | Xb |
| O | Xa | Xb | Xb | ✓ |
| G | ✓ | Xb | Xb | Xb |

47.

| | Coin | Stamp | Comic | Baseball |
|---|---|---|---|---|
| A | Xc | ✓ | Xc | Xd |
| C | Xb | Xd | Xa | ✓ |
| P | ✓ | Xc | Xc | Xb |
| S | Xb | Xc | ✓ | Xc |

48. a. Yes. Change the color of Iowa to yellow.

b. No. One possible explanation: Oklahoma, Arkansas, and Louisiana must each have a different color than the color of Texas and they cannot all be the same color. Thus, the map cannot be colored using only two colors.

49. home, bookstore, supermarket, credit union, home; or home, credit union, supermarket, bookstore, home.

50. home credit union, bookstore, supermarket, home.

51. N. These are the first letters of the counting numbers: **O**ne, **T**wo, **T**hree, etc. N is the first letter of the next number, which is **N**ine.

52. The symbols are formed by reflecting the numerals 1, 2, 3, … The next symbol would be a 6 preceded by a backward 6.

53. d is the correct choice. Example 3b found that quadrupling the length of a pendulum doubles its period. Doubling 1 second 4 times gives 16 seconds, which is close to the period of Foucault's pendulum. In order to double the period 4 times, we must quadruple the length of the pendulum 4 times.
$0.25 \times 4 \times 4 \times 4 \times 4 = 64$, so Foucault's pendulum should have a length of approximately 64 meters. Thus D, 67 meters, is the best choice.

54. a. 1010 is a multiple of 101, ($10 \times 101 = 1010$) but $11 \times 1010 = 11{,}110$. The digits of the product are not all the same.

b. For $n = 11$,
$n^2 - n + 11 = 11^2 - 11 + 11 = 121$,
which is not a prime number.

55. Answers will vary.

56. a. Answers will vary.

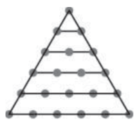
b. Most students find, by inductive reasoning, that the best strategy for winning the grand prize is to switch.



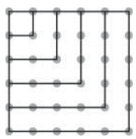
Full file at https://TestbankDirect.eu/

**EXCURSION EXERCISES, SECTION 1.2**

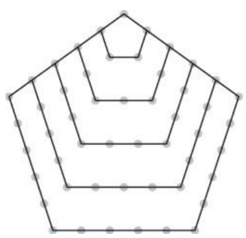
1. The sixth triangular number is 21.



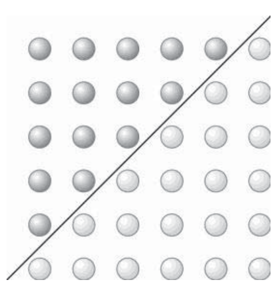
The sixth square number is 36.



The sixth pentagonal number is 51.



2. a. The fifth triangular number is 15. The sixth triangular number is 21. $15+21=36$, which is the sixth square number.

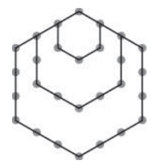


b. The $50^{th}$ triangular number is $\frac{50(50+1)}{2}=1275$. The $51^{st}$ triangular number is $\frac{51(51+1)}{2}=1326$. $1,275 + 1,326 = 2,601 = 51^2$, the $51^{st}$ square number.

c. The proof is:

$$\frac{n(n+1)}{2}+\frac{(n+1)(n+2)}{2}$$
$$=\frac{n+1}{2}(n+n+2)$$
$$=(n+1)^2$$

3. The fourth hexagonal number is 28.



**EXERCISE SET 1.2**

1.
1 7 17 31 49 71 97
6 10 14 18 22 26
4 4 4 4 4

26 + 71 = 97

2.
10 10 12 16 22 30 40
0 2 4 6 8 10
2 2 2 2 2

10 + 30 = 40

3.
–1 4 21 56 115 204 329
5 17 35 59 89 125
12 18 24 30 36
6 6 6 6

125 + 204 = 329

4.
0 10 24 56 112 190 280
10 14 32 56 78 90
4 18 24 22 12
14 6 –2 –10
–8 –8 –8

90 + 190 = 280

5.
9 4 3 12 37 84 159
–5 –1 9 25 47 75
4 10 16 22 28
6 6 6 6

75 + 84 = 159

6. 17 15 25 53 105 187 305
   −2 10 28 52 82 118
   12 18 24 30 36
   6 6 6 6

   118 + 187 = 305

7. Substitute in the appropriate values for $n$.

   For $n=1,\ a_1=\frac{1(2(1)+1)}{2}=\frac{3}{2}$

   For $n=2,\ a_2=\frac{2(2(2)+1)}{2}=5$

   For $n=3,\ a_3=\frac{3(2(3)+1)}{2}=\frac{21}{2}$

   For $n=4,\ a_4=\frac{4(2(4)+1)}{2}=18$

   For $n=5,\ a_5=\frac{5(2(5)+1)}{2}=\frac{55}{2}$

8. Substitute in the appropriate values for $n$.

   For $n=1,\ a_1=\frac{1}{1+1}=\frac{1}{2}$

   For $n=2,\ a_2=\frac{2}{2+1}=\frac{2}{3}$

   For $n=3,\ a_3=\frac{3}{3+1}=\frac{3}{4}$

   For $n=4,\ a_4=\frac{4}{4+1}=\frac{4}{5}$

   For $n=5,\ a_5=\frac{5}{5+1}=\frac{5}{6}$

9. Substitute in the appropriate values for $n$ to obtain 2, 14, 36, 68, 110.

10. Substitute in the appropriate values for $n$ to obtain 1, 12, 45, 112, 225.

11. Notice that each figure is square with side length $n$ plus an "extra row" of length $n-1$. Thus the $n$th figure will have $a_n=n^2+(n-1)$ tiles.

12. Start with a horizontal block of 2 tiles and one column of 3 tiles. Add a column of 3 tiles to each figure. Thus, the $n$th figure will have $a_n=3n+2$ tiles.

13. Each figure is composed of a horizontal group of $n$ tiles, a horizontal group of $n$ – 1 tiles, and a single "extra" tile. Thus the $n$th figure will have $a_n = n + n – 1 + 1 = 2n$ tiles.

14. Each figure is composed of a $(n + 2) \times (n + 2)$ square that is missing 1 tile.

    Thus the $n$th figure will have $a_n=(n+2)^2-1$ tiles.

15. a. There are 56 cannonballs in the sixth pyramid and 84 cannonballs in the seventh pyramid.

    b. The eighth pyramid has eight levels of cannonballs. The total number of cannonballs in the eighth pyramid is equal to the sum of the first 8 triangular numbers: 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 = 120.

16. Applying the formula:

$$Tetrahedra_{10}=\tfrac{1}{6}(10)(10+1)(10+2)$$
$$=\tfrac{1}{6}(10)(11)(12)$$
$$=\tfrac{1}{6}(1320)=220$$

17. a. Five cuts produce six pieces and six cuts produce seven pieces.

    b. The number of pieces is one more than the number of cuts, so $a_n=n+1$.

18. a. The difference table is shown
    2 4 7 11 16 22 29
    2 3 4 5 6 7
    1 1 1 1 1
    Seven cuts gives 29 pieces.

    b. The nth pizza-slicing number is one more than the nth triangular number.

19. a. Substituting in $n$ = 5:

$$P_5=\frac{5^3+5(5)+6}{6}=\frac{156}{6}=26$$

    b. Substituting several values:

$$P_6=\frac{6^3+5(6)+6}{6}=42<60$$
$$P_7=\frac{7^3+5(7)+6}{6}=64>60$$

    Thus the fewest number of straight cuts is 7.

20. a. Experimenting:
For $n > 2, 3F_n - F_{n-2} = F_{n+2}$
$n = 3 \Rightarrow 3F_3 - F_1 = 5 = F_5$
$n = 4 \Rightarrow 3F_4 - F_2 = 8 = F_6$
$n = 5 \Rightarrow 3F_5 - F_3 = 13 = F_7$
It appears as if this property is valid.

b. Experimenting:
$F_n F_{n+3} = F_{n+1} F_{n+2}$
$n = 2 \Rightarrow F_2 F_5 = 5 \neq 6 = F_3 F_4$
This property is not valid. The case $n = 2$ is a counterexample.

c. Experimenting:
$F_{3n}$ is an even number.
$n = 1 \Rightarrow F_3 = 2$
$n = 2 \Rightarrow F_6 = 8$
$n = 3 \Rightarrow F_9 = 34$
It appears as if this property is valid.

d. Experimenting:
For $n > 2,\ 5F_n - 2F_{n-2} = F_{n+3}$
$n = 3 \Rightarrow 5F_3 - 2F_1 = 8 = F_6$
$n = 4 \Rightarrow 5F_4 - 2F_2 = 13 = F_7$
$n = 5 \Rightarrow 5F_5 - 2F_3 = 21 = F_8$
It appears as if this property is valid.

21. Substituting:
$a_3 = 2 \cdot a_2 - a_1 = 10 - 3 = 7$
$a_4 = 2 \cdot a_3 - a_2 = 14 - 5 = 9$
$a_5 = 2 \cdot a_4 - a_3 = 18 - 7 = 11$

22. Substituting:
$a_3 = (-1)^3(3) + 2 = -1$
$a_4 = (-1)^4(-1) + 3 = 2$
$a_5 = (-1)^5(2) + (-1) = -3$

23. Substituting:
$F_{20} = 6765$
$F_{30} = 832,040$
$F_{40} = 102,334,155$

24. Substituting:
$F_{16} = 987$
$F_{21} = 10,946$
$F_{32} = 2,178,309$

25. The drawing shows the $n$th square number. The question mark should be replaced by $n^2$.

26. a. The new formula is:
$$a_n = 2\left[\frac{n(n-1)(n-2)(n-3)(n-4)}{4 \cdot 3 \cdot 2 \cdot 1}\right] + 2n$$

b. The new formula is:
$$a_n = 4\left[\frac{n(n-1)(n-2)(n-3)(n-4)}{4 \cdot 3 \cdot 2 \cdot 1}\right] + 2n$$

27. a. The eighth number is
(9.6)(2) + 0.4 = 19.2 + 0.4 = 19.6 AU
The ninth number is
(19.2)(2) + 0.4 = 38.4 + 0.4 = 38.8 AU

b. Bode's eighth number, 19.6 AU, is relatively close to the average distance from the Sun to Uranus.

c. Bode's ninth number, 38.8 AU, is not close (compared to the results obtained for the inner planets) to the average distance from the Sun to Neptune, which is 30.06 AU.

28. Bode's tenth number is (38.4)(2) + 0.4 = 76.8 + 0.4 = 77.2 AU, which is not close to Pluto's actual average distance from the sun of 39.44 AU.

29. No, but Bode's Law closely estimates the average distance of the first eight planets in our solar system.

30. a. $\frac{0.19}{0.46} \approx 0.41$ unit and $\frac{0.36}{0.46} \approx 0.78$ unit

b. The results are relatively close to Bode's first two numbers, 0.4 and 0.7.

c. It is interesting that the results compare so well with Bode's numbers, but we cannot say, based on this one example, that Bode's rule provides an accurate model for the placement of planets in other solar systems.

31. Answers may vary depending on the time of investigation.

32. Answers will vary.

33. a. For $n=1$, we get $1+2(1)+2=5=F_5$
For $n=2$, we get $1+2(2)+3=8=F_6$
For $n=3$, we get $2+2(3)+5=13=F_7$.
Thus, $F_n+2F_{n+1}+F_{n+2}=F_{n+4}$.

b. For $n=1$, we get $1+1+3=5=F_5$
For $n=2$, we get $1+2+5=8=F_6$
For $n=3$, we get $2+3+8=13=F_7$.
Thus, $F_n+F_{n+1}+F_{n+3}=F_{n+4}$.

34. a. For $n=2$, we get $1+1=2=F_4-1$
For $n=3$, we get $1+1+2=4=F_5-1$
For $n=4$, we get $1+1+2+3=7=F_6-1$.
For $n=5$, we get
$1+1+2+3+5=12=F_7-1$.
Thus, $F_1+F_2+\ldots+F_n=F_{n+2}-1$.

b. For $n=2$, we get $1+3=4=F_5-1$
For $n=3$, we get $1+3+8=12=F_7-1$
For $n=4$, we get
$1+3+8+21=33=F_9-1$.
Thus, $F_2+F_4+\ldots+F_{2n}=F_{2n+1}-1$

35. a.

| row | total |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |

Each row total is twice the number in the previous row. These numbers are powers of 2. It appears that the sum for the *n*th row is $2^n$. The sum of the numbers in row 9 is $2^9=512$.

b. They appear in the third diagonals.

36. Create a chart:

| Number of Days | Number of Pennies | How to find? |
|---|---|---|
| 1 | 1 | $2^1$-1 |
| 2 | 3 | $2^2$-1 |
| 3 | 7 | $2^3$-1 |
| 4 | 15 | $2^4$-1 |
| 5 | 31 | $2^5$-1 |
| 6 | 63 | $2^6$-1 |
| 7 | 127 | $2^7$-1 |
| 8 | 1255 | $2^8$-1 |
| 9 | 2511 | $2^9$-1 |
| 10 | 1023 | $2^{10}$-1 |
| 11 | 3047 | $2^{11}$-1 |
| 12 | 4095 | $2^{12}$-1 |
| 13 | 8191 | $2^{13}$-1 |
| 14 | 6383 | $2^{14}$-1 |
| 15 | 32,767 | $2^{15}$-1 |

a. 31 pennies or 31 cents

b. 1023 pennies or $10.23

c. 32,767 pennies or $327.67

d. By observing the pattern in the table above, the amount of money you would have in *n* days is $2^n$-1, where *n* equals the number of days.

37. a. 1 move

b. 3 moves

c. 7 moves (Start with the discs on post 1. Let A, B, C be the discs with A smaller than B and B smaller than C. Move A to post 2, B to post 3, A to post 3, C to post 2, A to post 1, B to post 2, and A to post 2.) This is 7 moves.

d. 15 moves

e. 31 moves

f. $2^n-1$ moves

g. $n = 64$, so there are $2^{64} - 1 = 1.849 \times 10^{19}$ moves required. Since each move takes 1 second, it will take $1.849 \times 10^{19}$ seconds to move the tower. Divide by 3600 to obtain the number of hours, then by 24 to obtain the days, then by 365 to obtain the number of years, about $5.85 \times 10^{11}$.

38. Using the hint and rearranging the equations:

$$F_n + F_{n-1} = F_{n+1}$$
$$F_n - F_{n-1} = F_{n-2}$$

Add the equations:

$$2F_n = F_{n+1} + F_{n-2}$$

Solve for $F_{n+1}$ to obtain $F_{n+1} = 2F_n - F_{n-2}$.

**EXCURSION EXERCISES, SECTION 1.3**

1. There is one route to point B, that of all left turns. Add the two numbers above point C to obtain 1 + 3 = 4 routes. Add the two numbers above point D to obtain 3 + 3 = 6 routes. Add the two numbers above point E to obtain 3 + 1 = 4 routes. As with point B, there is only one route to point F, that of all right turns.

2. Continue to fill in the numbers, adding the two numbers above each hexagon to obtain the number for that hexagon and labeling the first and last hexagon in each row with a one. The last row of numbers is 1, 7, 21, 35, 35, 21, 7, 1. There is only one route to point G. There are 1 + 7 = 8 routes to point H, 7 + 21 = 28 routes to point I, 21 + 35 = 56 routes to point J, and 35 + 35 = 70 routes to K.

3. The figure is symmetrical about a vertical line from A to K. Since J is the same distance to the left of the line AK as L is to the right of AK, the same number of routes lead from A to J as lead from A to L.

4. More routes lead to the center bin than to any of the other bins.

5. By adding adjacent pairs, the number of routes from A to P, Q, R, S, T, and U are 1, 9, 36, 84, 126, and 126, respectively.

**EXERCISE SET 1.3**

1. Let $g$ be the number of first grade girls, and let $b$ be the number of first grade boys. Then $b + g = 364$ and $g = b + 26$. Solving gives $g = 195$, so there are 195 girls.

2. Let $a$ be the length in feet of the shorter ladder and $b$ be the length in feet of the longer ladder. Then $a + b = 31.5$ and $a + 6.5 = b$. Solving gives $a = 12.5$ and $b = 19$, so the ladders are 12.5 feet and 19 feet long.

3. There are 36 $1 \times 1$ squares, 25 $2 \times 2$ squares, 16 $3 \times 3$ squares, 9 $4 \times 4$ squares, 4 $5 \times 5$ squares and 1 $6 \times 6$ square in the figure, making a total of 91 squares.

4. The first decimal digit, like all the odd decimal digits, is a zero, and the second decimal digit, like all the even decimal digits, is a 9. Since 44 is even, the 44th decimal digit is a 9.

5. Solving:

$$x = \text{cost of the shirt}$$
$$x - 30 = \text{cost of the tie}$$
$$(x - 30) + x = 50$$
$$2x - 30 = 50$$
$$2x = 80$$
$$x = 40$$

The shirt costs $40.

6. Using the results of example 3, 12 teams play each of 11 teams for a total of $(12 \times 11) \div 2 = 66$ games. Since each team plays each of the teams twice $2 \times 66 = 132$ total games.

7. There are 14 different routes to get to Fourth Avenue and Gateway Boulevard and 4 different routes to get to Second Avenue and Crest Boulevard. Adding gives that there are 18 different routes altogether.

8. a. There are 2 different routes from point A to the Starbucks and 2 different routes from the Starbucks to point B. Multiplying gives a total of 4 different routes.

   b. There is only one direct route to the Subway. There are 3 different routes from the Subway to point B, so there are 3 different routes altogether.

c. Since there is only one direct route to the Subway, starting the count there will not change the number of routes. There is only one direct route from the Subway to Starbucks, and there are 2 different routes from Starbucks to point B, so there are 2 different routes altogether.

9. Try solving a simpler problem to find a pattern. If the test had only 2 questions, there would be 4 ways. If the test had 3 questions, there would be 8 ways. Further experimentation shows that for an $n$ question test, there are $2^n$ ways to answer. Letting

$n = 12$, there are $2^{12} = 4,096$ ways

10. The frog gains 2 feet for each leap. By the 7th leap, he has gained 14 feet. On the 8th leap, he moves up 3 feet to 17 feet, escaping the well.

11. 8 people shake hands with 7 other people. Multiply 8 and 7 and divide by 2 to eliminate repetitions to obtain 28 handshakes.

12. There are $\frac{24\times 23}{2} = 276$ ways to join the points.

13. Let $p$ be the number of pigs and let $d$ be the number of ducks. Then $p + d = 35$ and $4p + 2d = 98$. Solving gives $d = 21$ and $p = 14$, so there are 21 ducks and 14 pigs.

14. Carla arrives home first because she spends more time running than does Allison.

15.

| Dimes | Nickels | Pennies |
|---|---|---|
| 0 | 0 | 25 |
| 0 | 1 | 20 |
| 0 | 2 | 15 |
| 0 | 3 | 10 |
| 0 | 4 | 5 |
| 0 | 5 | 0 |
| 1 | 0 | 15 |
| 1 | 1 | 10 |
| 1 | 2 | 5 |
| 1 | 3 | 0 |
| 2 | 0 | 5 |
| 2 | 1 | 0 |

There are 12 ways.

16. Area of the room is $12 \times 15 = 180$ square feet. Each square of carpet has an area 9 square feet. Divide 180 by 9 to get 20 squares.

17. The units digits of powers of 4 form the sequence 4, 6, 4, 6, . . . . Even powers end in 6. Therefore, the units digit of $4^{300}$ is 6.

18. The units digits of powers of 2 form the sequence 2, 4, 8, 6, 2, 4, 8, 6. . . . Divide 725 by 4 to obtain a remainder of 1 which corresponds to 2. Therefore the units digit of $2^{725}$ is 2.

19. The units digits of powers of 3 form the sequence 3, 9, 7, 1, 3, 9, 7, 1, . . . . Divide 412 by 4 to obtain the remainder 0, which corresponds to 1. Therefore the units digit of $3^{412}$ is 1.

20. The units digits of powers of 7 are 7, 9, 3, 1, 7, 9, 3, 1, …. Divide 146 by 4 to obtain the remainder 2, which corresponds to 9. Therefore the units digit of $7^{146}$ is 9.

21. a. Add the numbers in pairs: 1 and 400, 2 and 399, 3 and 398, and so on. There are 200 pair sums equal to 401.
$200 \times 401 = 80,200$.

b. Add the numbers in pairs: 1 and 550, 2 and 549, 3 and 548 and so on. There are 275 pair sums equal to 551.
$275 \times 551 = 151,525$.

c. Add the numbers in pairs: 2 and 84, 4 and 82, and so on, leaving off the 86. There are 21 sums of 86 plus one additional 86.
$21 \times 86 + 86 = 1,892$.

22. One method is to apply the procedure used by Gauss to find the sum of the numbers from 1 to 64 and then add 65 to this total.

23. a. 121, 484, and 676 are the only three-digit perfect square palindromes.

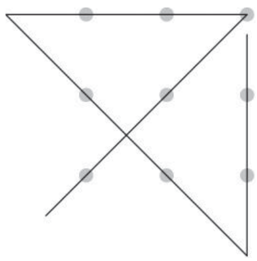
b. 1331 is the only four-digit perfect cube palindrome.

24. The next palindromic number is 16061. The distance traveled is $16061 - 15951 = 110$ miles. The average speed is $110/2 = 55$ mph.

25. Draw a simpler picture:


Note that the first page of the first volume is the second dot on the line, and the last page of the third volume is the seventh dot on the line. This is because when books sit on a shelf, the first pages are on the right side of the book and their last pages are on the left side.
$\frac{1}{8}+\frac{1}{8}+1+\frac{1}{8}+\frac{1}{8}=1\frac{1}{2}$ inches.

26. Yes, it is possible. One possible way is shown below.



27. a. 1.4 billion

b. 2008

c. 2002

28. a. 35 - 54

b. 103,085,520(0.253) ≈ 26,080,000

29. a. 2005

b. 2009

c. The number of ticket buyers was less.

30. Trying several values for the number of voters, $n$:

If $n=10$, the number of votes must be between 9.4 and 10. If $n=15$, the number of votes must be between 14.1 and 15. If $n=16$, the number of votes must be between 15.04 and 16. If $n=17$, the number of votes must be between 15.98 and 17. It is possible to have 16 votes for the candidate. Thus, the least possible number of votes cast is 17.

31. Since there is one blue tile in each column, there are $n$ blue tiles on the diagonal that starts in the upper left hand corner. Similarly, there are $n$ blue tiles in the diagonal that starts in the upper right hand corner. The two diagonals have one tile in common, so the actual total number of blue tiles is $2n-1$. Since $2n-1=101$, we can solve to find $n=51$. The total number of tiles is $n^2$. Substituting the value for $n$ yields 2601.

32. Let $b$ be the number of boys in the family and $g$ be the number of girls. If each girl has as many brothers as sisters, $b=g-1$. If each boy has twice as many sisters as brothers, $g=2(b-1)$. Substituting for $b$ in the second equation, we get $g=2(g-2)$. Solving, $g=4$ and $b=3$. Thus, there are 7 children.

33. Let $b$ be the number of boys in the family and $g$ be the number of girls. The first two statements imply that the speaker is a girl. Thus, $g-1=b+2$. Solving for $b$, $b=g-3$. To answer the last question, we must omit the youngest brother, so $b-1=g-4$. There are four more sisters than brothers.

34. If you take 22 pennies, you have 22 pennies.

35. The bacteria population doubles every day, so on the $11^{th}$ day there are half as many bacteria as on the $12^{th}$ day.

36. a. Let A, B, C, and D represent the four people with weights 80, 100, 150, and 170 pounds, respectively. A and B make the first trip across. A comes back alone. C crosses the river and B comes back alone. C is now on the opposite bank. Repeat this procedure to get D to the opposite bank. Then one more trip will get both A and B to the opposite bank.

b. The minimum number of crossings is 9.

37. Let $x$ be the score that Dana needs on the fourth exam:

$$\frac{82+91+76+x}{4}=85$$
$$249+x=340$$
$$x=91$$

38. Fill the 5-gallon jug. Pour from it into the 3-gallon jug, until the 3-gallon jug is full. Now empty the 3-gallon jug. There are 2 gallons in the 5-gallon jug. Pour these 2 gallons into the 3-gallon jug. Fill the 5-gallon jug and pour from it into the 3- gallon jug. At this point there are 2 gallons in the 3-gallon jug, so only 1 more gallon will fit. Thus, when the 3-gallon jug is full, there will be 4 gallons left in the 5-gallon jug.

39. a. Place four coins on the left balance pan and the other 4 coins on the right balance pan. The pan that is the higher contains the fake coin. Take the four coins from the higher pan and use the balance scale to compare the weight of two of these coins to the weight of the other two coins. The pan that is the higher contains the fake coin. Take the two coins from the higher pan and use the balance scale to compare the weights. The pan that is the higher contains the fake coin. This procedure enables you to determine the fake coin in 3 weighings.

    b. Place 3 of the coins on one of the balance pans and 3 coins on the other balance pan. If the pans balance, then the fake coin is one of the two remaining coins. You can put each one of these coins on a balance pan and the higher pan contains the fake coin. If the 3 coins on the left do not balance with the 3 coins on the right, then the higher pan contains the fake coin. Pick any 2 of these 3 coins and use the balance scale to compare their weights. If these 2 coins do not balance, then the higher pan contains the fake coin. If these two coins balance, then the 3rd coin (the one that you did not place on the balance pan) is the fake. In either case this procedure enables you to determine the fake coin in 2 weighings.

40. The correct answer is c., 21:00. If it were two hours later (23:00 – 1 hour before midnight), it would be half as long as if it were an hour later (22:00 – 2 hours before midnight).

41. The correct answer is a. Sally likes perfect squares.

42. The correct answer is b. The other 800 elephants can be any mix of all blue and pink and green stripes.

43. The correct answer is d. The numbers are all perfect cubes. The missing number is the cube of 4.

44. a. Evaluating:

$$3^{\left(3^3\right)} = 3^{27}$$

$$\left(3^3\right)^3 = 3^9$$

$$3^{27} \div 3^9 = 3^{18}$$

It is $3^{18}$ times as large.

    b. Evaluating:

$$4^{\left(4^4\right)} = 4^{256}$$

$$\left(4^4\right)^4 = 4^{16}$$

$$4^{256} \div 4^{16} = 4^{256-16} = 4^{240}$$

It is $4^{240}$ times as large.

45. a. Write an equation. When the people who were born in 1980 are $x$ years old, it will be the year $1980 + x$. We are looking for the year that satisfies $1980 + x = x^2$. Solving gives $x = 45$ and $x = -44$. The solution must be a natural number, so $x = 45$. Therefore when the people born in 1980 are 45 years old, the year will be $45^2 = 2025$.

    b. 2070, because people born in 2070 will be 46 in $2116 = 46^2$.

46. Adding 83 is the same as adding 100 and subtracting 17. Thus, after you add 83, you will have a number that has 1 as the hundreds digit. The number formed by the tens digit and the units digit will be 17 less than your original number. After you add the hundreds digit, 1, to the other two digits of this new number, you will have a number that is 16 less than your original number. If you subtract this number from your original number, you must get 16.

47. It takes 9 1-digit numbers for pages 1-9, 180 digits for pages 10-99, and 423 digits for pages 100-240. The total is 612.

48.

| | | | | | |
|---|---|---|---|---|---|
| 6 | 2 | **3** | **4** | 5 | **1** |
| **5** | **1** | 4 | 3 | **2** | **6** |
| **3** | 6 | **2** | 5 | **1** | 4 |
| **4** | **5** | 1 | **6** | 3 | **2** |
| 1 | **3** | 6 | 2 | **4** | 5 |
| **2** | 4 | **5** | 1 | 6 | **3** |

49. Answers will vary.

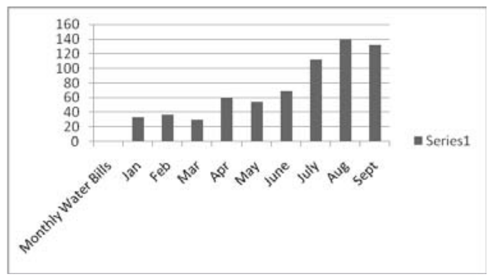
50. M = 1, S = 9, E = 5, N = 6, D = 7, O=0, R = 8, Y = 2

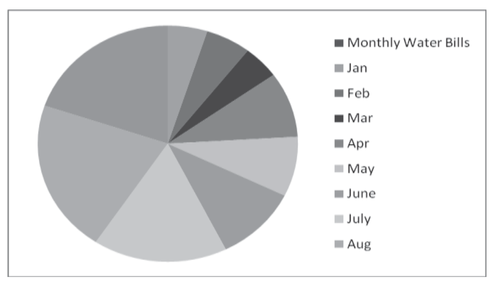
51. a. The Collatz problem (the 3*n* + 1 problem): Start with any counting number greater than 1. Now generate a sequence of numbers using the following rules; If *n* is even, divide *n* by 2. If *n* is odd, multiply *n* by 3 and add 1. Repeat the above procedure on the new number you have just generated. Keep applying the above procedure until you obtain the number 1.
Collatz conjectured that the procedure would always generate a sequence of numbers that would eventually reach the number 1, regardless of the starting number n. Thus far no one has been able to to prove that Collatz's conjecture is true or to show that it is false. The sequences are sometimes called "hailstone" sequences because the numbers in the sequence tend to bounce up and down, much like a hailstone in a storm.

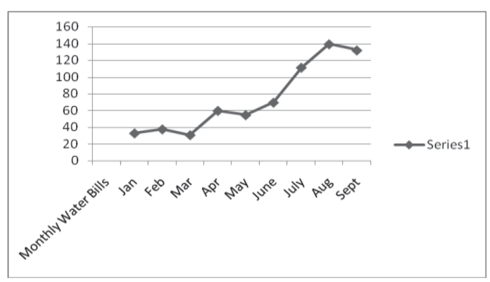
b. Here are the Collatz sequences for the counting numbers 2 through 10.
2 : 2, 1
3: 3, 10, 5, 16, 8, 4, 2, 1
4: 4, 2, 1
5: 5, 16, 8, 4, 2, 1
6: 6, 3, 10, 5, 16,8, 4, 2, 1
7: 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
8: 8, 4, 2, 1
9: 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
10: 10, 5, 16, 8, 4, 2, 1

52. Reports should include information on Erdos's love of mathematics and his passion for working on unsolved problems. Erdos was one of the great mathematicians of the twentieth century, and one of the most prolific. Erdos loved difficult problems that were easy to state and understand without learning a lot of definitions. Much of his work was in the area of combinatorics and number theory. Concerning the Collatz problem Erdos stated, "Mathematics is not yet ready for such problems."

53. a.




b. Circle Graph

54. Responses will vary about the person's life.

## CHAPTER 1 REVIEW EXERCISES

1. This argument reaches a conclusion based on a case of a general assumption, so it is an example of deductive reasoning.

2. This argument reaches a conclusion based on specific examples, so it is an example of inductive reasoning.

3. This argument reaches a conclusion based on a specific example, so it is an example of inductive reasoning.

4. This argument reaches a conclusion based on a case of a general assumption, so it is an example of deductive reasoning.

5. Any number from 0 to 1 provides a counterexample. For example, $x = \frac{1}{2}$ provides a counterexample because $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ and $\frac{1}{16}$ is not greater than $\frac{1}{2}$.

6. $n = 4$ provides a counterexample because $\frac{(4)^3 + 5(4) + 6}{6} = \frac{90}{6} = 15$, which is not even.

7. $x = 1$ provides a counterexample because $(1+4)^2 = (5)^2 = 25$ and $1^2 + 4^2 = 1 + 16 = 17$

8. $a = 1$ and $b = 1$ provides a counterexample because $(1+1)^3 = 2^3 = 8$, but $1^3 + 1^3 = 1 + 1 = 2$.

9. a. –2 2 12 28 50 78 **112**
   4 10 16 22 28 34
   6 6 6 6 6

   Add 34 and 78 to obtain 112.

   b. –4 –1 14 47 104 191 314 **479**
   3 15 33 57 87 123 165
   12 18 24 30 36 42
   6 6 6 6 6

   Add 165 and 314 to obtain 479.

10. a. 5 6 3 – 4 –15 –30 –49 **–72**
    1 –3 –7 –11 –15 –19 –23
    –4 –4 –4 –4 –4 –4

    Add –23 and –49 to obtain –72.

    b. 2 0 –18 –64 –150 –288 –490 **–768**
    –2 –18 –46 –86 –138 –202 –278
    –16 –28 –40 –52 –64 –76
    –12 –12 –12 –12 –12

    Add –278 and –490 to obtain –768.

11. Substituting:

$$a_1 = 4(1)^2 - 1 - 2 = 4 - 3 = 1$$
$$a_2 = 4(2)^2 - 2 - 2 = 16 - 4 = 12$$
$$a_3 = 4(3)^2 - 3 - 2 = 36 - 5 = 31$$
$$a_4 = 4(4)^2 - 4 - 2 = 64 - 6 = 58$$
$$a_5 = 4(5)^2 - 5 - 2 = 100 - 7 = 93$$
$$\begin{aligned} a_{20} &= 4(20)^2 - 20 - 2 \\ &= 4(400) - 20 - 2 \\ &= 1600 - 22 = 1578 \end{aligned}$$

12. $F_7 = F_6 + F_5 = 8 + 5 = 13$
    $F_8 = F_7 + F_6 = 13 + 8 = 21$
    $F_9 = F_8 + F_7 = 21 + 13 = 34$
    $F_{10} = F_9 + F_8 = 34 + 21 = 55$
    $F_{11} = F_{10} + F_9 = 55 + 34 = 89$
    $F_{12} = F_{11} + F_{10} = 89 + 55 = 144$

13. Each figure has a horizontal section with $n+1$ tiles, a horizontal section with $n$ tiles, and a vertical section with $n-1$ tiles.
    $a_n = n + 1 + n + n - 1 = 3n$

14. Each figure is a square with side length $n+2$ and $n$ tiles removed.
$$\begin{aligned} a_n &= (n+2)^2 - n = n^2 + 4n + 4 - n \\ &= n^2 + 3n + 4 \end{aligned}$$

15. Each figure is a square with side length $n+1$ with an attached diagonal with $n+1$ tiles.

$$a_n = (n+1)^2 + (n+1) = n^2 + 3n + 2$$

16. Each figure made up of four sides of length $n$ with a diagonal piece in the middle with length $n-1$.

$$a_n = 4n + (n-1) = 5n - 1$$

17. Let $x$ be the width. Then $5x$ is the length. Since one length already exists, only 3 sides of fencing are needed. The total perimeter is $5x + x + x = 2240$. Solving, we find $x = 320$. The dimensions are 320 ft. by 1600 ft.

18. Solve a simpler problem. If the test has 1 question, there are 3 ways to answer. If the test has 2 questions, there are 9 ways to answer. If the test has 3 questions, there are 27 ways to answer. It appears that for a test with $n$ questions, there are $3^n$ ways to answer. In this case, $n$ = 15, so there are $3^{15} = 14,348,907$ ways to answer the test.

19. If the 11th and 35th are opposite each other, there must be 23 more skyboxes between them (going in each direction). The total is $23 + 1 + 23 + 1 = 48$.

20. On the first trip the rancher takes the rabbit across the river. The rancher returns alone. The rancher takes the dog across the river and returns with the rabbit. The rancher next takes the carrots across the river and returns alone. On the final trip the rancher takes the rabbit across the river.

21. $1400 − $1200 = $200 profit.
$1900 − $1800 = $100 profit.
Total profit = $200 + $100 = $300

22. Multiply 15 and 14 and divide by 2 to eliminate repetitions. 105 handshakes will take place.

23. Answers will vary. Possible answers include: make a list, draw a diagram, make a table, work backwards, solve a simpler similar problem, look for a pattern, write an equation, perform an experiment, guess and check, and use indirect reasoning.

24. Answers will vary. Possible answers include: ensure that the solution is consistent with the facts of the problem, interpret the solution in the context of the problem, and ask yourself whether there are generalizations of the solution that could apply to other problems.

25.

| | CS | Chem | Bus | Bio |
|---|---|---|---|---|
| M | Xa | Xd | Xd | ✓ |
| C | Xd | Xb | ✓ | Xb |
| R | ✓ | Xb | Xd | Xb |
| E | Xd | ✓ | Xd | Xc |

26.

| | Bank | Super | Service | Drug |
|---|---|---|---|---|
| D | Xd | Xd | Xd | ✓ |
| P | Xb | ✓ | Xd | Xb |
| T | ✓ | Xd | Xa | Xd |
| G | Xb | Xc | ✓ | Xb |

27. a. Yes. Answers will vary.

b. No. The countries of India, Bangladesh, and Myanmar all share borders with each of the other two countries. Thus, at least three colors are needed to color the map.

28. a. If we label the three islands A, B and C from left to right, the following sequence of moves shows a route that starts at North Bay and passes over each bridge once and only once. Starting on the North Bay, cross the bridge to Island A then cross the bridge to the South Bay. Travel to the right on South Bay and cross the bridge back to Island A, then cross the bridge to Island B. From here, cross to the North Bay and travel right on North Bay to cross over to Island C. From here, cross to Island B, then to the South Bay. Travel right on South Bay, crossing to Island C and finally to Island B.

b. No.

29. Draw the three possible pictures (one with $x$ diagonal from 2, one diagonal from 5 and one diagonal from 10) to find the three possible values for $x$: 1 square inch, 4 square inches, 25 square inches.

30. a. Adding smaller line segments to each end of the shortest line doubles the total number of line segments. Thus the nth figure has $2^n$ line segments. For $n$ = 10, $a_{10} = 1024$.

b. $a_{30} = 2^{30} = 1,073,741,824$

31. A = 1, B = 9, D = 0

32. Making a table:

| quarters | nickels |
|---|---|
| 4 | 0 |
| 3 | 5 |
| 2 | 10 |
| 1 | 15 |
| 0 | 20 |

There are 5 ways.

33. Use a list. WWWLL, WWLWL, WWLLW, WLWLW, WLLWW, WLWWL, LWWWL, LWLWW, LWWLW, LLWWW.
There are 10 ways.

34. The units digit of powers of 7 are 7, 9, 3, 1, 7, 9, 3, 1, …. Divide 56 by 4 to obtain the remainder of 0, which corresponds to 1. Therefore the units digit of $7^{56}$ is 1.

35. The units digit of powers of 23 are 3, 9, 7, 1, 3, 9, 7, 1, …. Divide 85 by 4 to obtain the remainder of 1, which corresponds to 3. Therefore the units digit of $23^{85}$ is 3.

36. Pick a number $n$:

$n$

$4n$ multiply by 4

$4n+12$ add 12

$\frac{4n+12}{2}=2n+6$ divide by 2

$2n+6-6=2n$ subtract 6

37. 2000 nickels are equivalent to \$100.00. 2004 nickels is \$0.20 more than \$100 or \$100.20.

38. a. \$3.27

b. 2007 - 2008

39. a. $(9.089)(0.651)\approx 5.9$ billion

b. Yahoo: $(9.089)(0.135)\approx 1.2270$
AOL: $(9.089)(0.025)\approx 0.2272$
$\frac{1.2270}{0.2272}\approx 5.4$ billion times

40. a. 2.7 million dollars

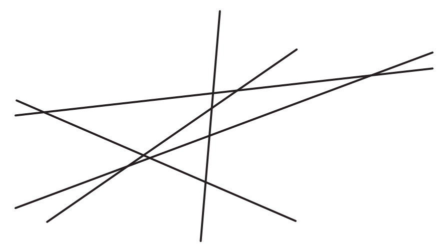
b. 207 – 2008, 2008 – 2009

c. 3 ÷ 98 = 0.03, 3 cents per viewer

41. Every multiple of 5 ends in a 5 or a zero. Every palindromic number begins with the same digit it ends with. We cannot begin a number with 0, so the number must end in 5. Thus it must begin with 5. The smallest such number is 5005.

42. Checking all of the two-digit natural numbers shows that there are no narcissistic numbers.

43. a. 10 intersections.



b. Yes

44. Construct a difference table as shown below.

2 6 12 20 **30**
4 6 8 **10**
2 2 **2**

The second differences are all the same constant, 2. Extending this row so that it includes additional 2 enables us to predict that the next first difference will be 10. Adding 10 to the fourth term 20 yields 30. Using the method of extending the difference table, we predict that 30 is the next term in the sequence.

45. a. 22. $9^{22}$ has 21 digits.

b. $9^9=387,420,489$, so $9^{(9^9)}$ is the product of 387,420,489 nines. At one multiplication per second this would take about 12.3 years. It is probably not a worthwhile project.

**CHAPTER 1 TEST**

1. This argument reaches a conclusion based on a specific example, so it is an example of inductive reasoning.
2. This conclusion is based on a specific case of a general assumption, so this argument is an example of deductive reasoning.

3. –1 0 9 32 75 144 245 **384**
   1 9 23 43 69 101 **139**
   8 14 20 26 32 **38**
   6 6 6 6 **6**

   Add 139 and 245 to obtain 384.

4. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

5. a. Each figure contains a horizontal group of $n$ tiles, a horizontal group of $n$ + 1 tiles, and a vertical group of $n$ – 1 tiles.
   $a_n = n + n + 1 + 2n - 1 = 4n.$

   b. Each figure contains a horizontal group of $n$ + 1 tiles and 2 horizontal groups of $n$ tiles.
   $a_n = n + 1 + n + n = 3n + 1.$

6. $a_1 = 0.\ a_2 = 1.\ a_3 = -3.\ a_4 = 6.\ a_5 = -10.$
   $$a_{105} = (-1)^{105}\left(\frac{105\,(104)}{2}\right)$$
   $$= -1(105 \cdot 52) = -5,460$$

7. $a_3 = 2a_{3-1} + a_{3-2}$
   $= 2a_2 + a_1$
   $= 2(7) + 3$
   $= 14 + 3$
   $= 17$
   $a_4 = 2a_{4-1} + a_{4-2}$
   $= 2a_3 + a_2$
   $= 2(17) + 7$
   $= 34 + 7$
   $= 41$
   $a_5 = 2a_{5-1} + a_{5-2}$
   $= 2a_4 + a_3$
   $= 2(41) + 17$
   $= 82 + 17$
   $= 99$

8. a. Construct a difference table as shown below.

   0 2 5 9 **14**
   2 3 4 **5**
   1 1 **1**

   The second differences are all the same constant, 1. Extending this row so that it includes additional 1 enables us to predict that the next first difference will be 5. Adding 5 to the fourth term 9 yields 14. Using the method of extending the difference table, we predict that 14 is the next term in the sequence.

   b. Construct a difference table as shown below.

   0 2 5 9 14 **20**
   2 3 4 5 **6**
   1 1 1 **1**

   The second differences are all the same constant, 1. Extending this row so that it includes additional 1 enables us to predict that the next first difference will be 6. Adding 6 to the fifth term 14 yields 20. Using the method of extending the difference table, we predict that 20 is the next term in the sequence.

9. Understand the problem. Devise a plan. Carry out the plan. Review the solution.

10. Making a table:

| Half-dollars | Quarters | Dimes |
|---|---|---|
| 2 | 0 | 0 |
| 1 | 0 | 5 |
| 1 | 2 | 0 |
| 0 | 4 | 0 |
| 0 | 2 | 5 |
| 0 | 0 | 10 |

There are 6 ways.

11. Make a list.
LLWWWW LWLWWW LWWLWW
LWWWLW LWWWWL WLWWWL
WLWWLW WLWLWW WLLWWW
WW LLWW WWWLWL WWWWLL
WWLWWL WWWLLW WWLWLW
There are 15 ways.

12. The units digits form a sequence with 4 terms that repeat, so divide the powers by 4 and look at the remainders. A remainder of 1 corresponds to a 3.

13. Work backwards. Subtract \$150 from \$326. This is \$176. Let $x$ be the amount of money Shelly had before renting the room.

    Then $x - \frac{x}{3} = \$176$, so $x$ = \$264. Adding on \$22 and \$50 gives \$336. Since this is half of her savings (the other half was spent on the plane ticket), double to get \$672.

14. 126 ways. Add successive pairs of vertices. The last pair give 70 ways + 56 ways or 126 ways.

15. Multiply 9 times 8 and divide by 2 to eliminate repetitions. There will be a total of 36 league games.

16.

| | 5 | 7 | 13 | 15 |
|---|---|---|---|---|
| Rey | Xa | Xc | ✓ | Xd |
| Ram | ✓ | Xc | Xc | Xc |
| Shak | Xc | Xc | Xd | ✓ |
| Sash | Xc | ✓ | Xc | Xb |

17. $x = 4$ gives $\frac{(4-4)(4+3)}{(4-4)} = \frac{0}{0}$, which makes the left side of the equation meaningless since division by zero is undefined, but in any case not equal to 4 + 3 = 7.

18. a. 2003

    b. 1,096,000 – 957,000 = 139,000 more